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Numerical study of conformal invariance in RSOS models

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Abstract. I study conformal invariance in the recently solved RSOS models of Andrews *et al* at both the ferromagnetic and antiferromagnetic transitions, using finite-size scaling of the transfer matrix. I confirm the conjecture of Huse concerning the conformal classification of the ferromagnetic transitions and give evidence that the antiferromagnetic transitions lie in the universality class of Z_n models. In addition, I show that conformal results can be applied to anisotropic systems if logarithmic corrections are taken into account.

1. Introduction

In the past few years, a great deal of progress has been made in understanding the role of conformal invariance in two-dimensional statistical mechanics systems at criticality. The reason that two-dimensional systems are of special interest is that here the conformal transformations form an infinite-dimensional group, in contrast to the higher-dimensional analogues. This has the result that the correlation functions are greatly restricted if one postulates conformal invariance.

A parameter of special importance in these theories is the central charge of the Virasoro algebra, which enters as a multiplicative constant in the two-point correlation function of the stress-energy tensor of the Lagrangian field theory associated with the continuum limit of the critical system. A major reason for the importance of this parameter comes from the FQS classification scheme of Freidan *et al* [1]. In this scheme, if the value of c is less than 1 and the theory is unitary in the field-theoretic sense, then c must fall into a discrete series between $\frac{1}{2}$ and 1. Moreover, once c is known, the possible scale dimensions of the theory are restricted to a small discrete series. Thus, if we know the central charge of a model, and it falls into the above classification, then the values of the critical exponents are also known. For c greater than 1, the classification does not seem to be useful except in models with supersymmetry.

By a remarkable coincidence, at about the same time that the above developments were taking place, a perfect laboratory for conformal invariance was solved exactly (for free energy and a subset of the critical exponents) by Andrews *et al* [2]. These models—generalisations of the hard square lattice gas with from two to an infinite number of states—are called restricted solid-on-solid (RSOS) models. The different models are parametrised by a number r which takes integer values from 4 to ∞.

The exact solution located two two-dimensional surfaces of criticality in the parameter space. Huse [3] studied a subset of the scale dimensions of these two transitions

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predicted by the exact solution and interpreted them from the standpoint of conformal invariance. The first transition, antiferromagnetic in nature, had scale dimensions which did not fit into the FQS classification and, because the model has a Hermitian transfer matrix (it is unitary), one conjectures that c is greater than 1 or the model has no conformal invariance. In addition, it has been noticed that the subset of critical exponents computed for this model match those of the recently solved Z_n models [4] (which have c greater than 1). The second transition had scale dimensions which were consistent with a subset of those predicted by the FQS classification, which Huse used to conjecture that this transition represents a generic example of the multicritical points in the FQS classification.

Until now these statements have remained conjectures because the central charge has neither been computed nor measured numerically except in the case $r=4$, where the model reduces to the Ising model. The fact that it has not been measured does not seem too surprising because the appropriate technique would be finite-size scaling of the transfer matrix or Hamiltonian, with the transfer matrix preferred for its generality, and the difficulty of using such techniques increases extremely rapidly with the number of states available to the variables of the model. However, the many-state nature of the RSOS models is somewhat deceptive, and in the third section of this paper, an upper bound will be given on the number of states of the system on a finite lattice, which implies that the computational expense of models with reasonably low values of r is small. It is shown in the next section that it is possible to study models up to $r=12$ as cheaply as the Ising model.

In § 2, I describe briefly the RSOS models and give the Boltzmann weights on the critical surfaces as found by Andrews *et al* [2]. In § 3, finite-size scaling results are presented for the ferromagnetic transition. I find the conjecture of Huse is borne out by numerical measurement. Also in § 3, I study the effect of anisotropy on the relation between finite-size scaling amplitudes, scale dimensions and central charge for the case $r=4$. I find that the logarithmic corrections due to the marginal anisotropy operator can be fitted into a scheme to treat marginal operators recently proposed by Cardy [5]. This picture is necessary for the study of the antiferromagnetic transition. In § 4, the antiferromagnetic transition is studied. I find that the Virasoro charge as computed by the row-to-row transfer matrix with periodic boundary conditions does not have a good thermodynamic limit, as expected from [2]. However, the physics of the situation dictates a solution to this problem and, within the rather large errors forced by slow convergence of estimates to c, I find consistency with the transitions being in the same universality class as Z_n transitions. In § 5, conclusions are presented. Finally, some computational details are presented in an appendix.

2. Description of RSOS models

These models are parametrised by an integer r taking values from 4 to ∞. The variables of the lattice can take values from 1 to $r-1$. In addition, variables at neighbouring sites must differ by exactly one. $r=4$ is the Ising model. The partition function of the model is

$$Z=\sum_{\text{conf}}\prod_{\text{plaq}} W(\text{plaq}) \tag{1}$$

where the Boltzmann weights W depend on the four sites on the corners of the

plaquette. Andrews *et al* [2] write these Boltzmann weights as follows:

$$W(l, l+1; l-1, l) = W(l, l-1; l+1, l) = \alpha(l) \tag{2a}$$

$$W(l+1, l; l, l-1) = W(l-1, l; l, l+1) = \beta(l) \tag{2b}$$

$$W(l+1, l; l, l+1) = \gamma(l) \tag{2c}$$

$$W(l-1, l; l, l-1) = \delta(l). \tag{2d}$$

Figure 1 shows the correspondence between the arguments of the Boltzmann weights and the positions on the plaquette.

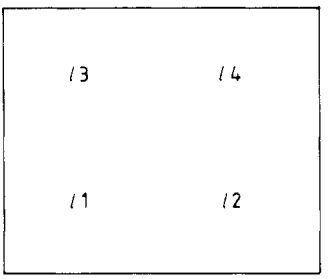

Figure 1. Notation for $W(l1, l2; l3, l4)$.

The expressions for α, β, γ and δ are elliptic functions of the parameters of the model and I do not reproduce them here. Instead, I write the weights at criticality, which are simple trigonometric functions:

$$\alpha(l) = \sin(v+\eta) \tag{3a}$$

$$\beta(l) = \sin(\eta - v)\{\sin[2\eta(l-1)]\sin[2\eta(l+1)]\}^{1/2}/\sin(2\eta l) \tag{3b}$$

$$\gamma(l) = \sin(2\eta)\sin[(2l+1)\eta - v]/\sin(2l\eta) \tag{3c}$$

$$\delta(l) = \sin(2\eta)\sin[(2l-1)\eta + v]/\sin(2l\eta) \tag{3d}$$

where η is given by

$$\eta = \pi/2r \tag{4}$$

and, in comparison with equation (1.2.1) of [2], w_0 has been set to zero and $\rho' = \frac{1}{2}p^{1/4}$. Here the parameter v measures the anisotropy of the model with $v = 0$ isotropic.

It is simple to see that, because of the restriction that neighbouring lattice sites differ in value by exactly one, the lattice divides into two interpenetrating lattices with variables odd on one of the lattices and even on the other.

The above weights define the critical surface of the model; it is one dimensional, parametrised by v. There is a further division of the parameter space into four regimes. In the language of Andrews *et al*, the transition from regime I to II occurs for $\eta < v < 3\eta$ and is of antiferromagnetic nature, while the transition from regime III to IV occurs for $-\eta < v < \eta$.

Freidan *et al* [1] have classified unitary two-dimensional models with conformal invariance and have shown that if c, the central charge of the Virasoro algebra, is less than one then it can have only the values

$$c = 1 - 6/m(m+1) \qquad m = 3, 4, \ldots \tag{5}$$

with m an integer. In addition, all possible scale dimensions of the theory are given by the series

$$h_{p,q}(m)=\frac{[(m+1)p-mq]^2-1}{4m(m+1)} \qquad p=(1,\ldots,m-1),\ q=(1,\ldots,m). \tag{6}$$

In addition to the understanding of two-dimensional critical phenomena provided by these relations, we are given a practical classification scheme for the universality classes of two-dimensional statistical mechanics models.

Based on the subset of scale dimensions of the RSOS model provided by the exact solution of [2], Huse conjectured, for the III–IV transition, that

$$m=r-1 \tag{7}$$

while the I–II transitions had scale dimensions which did not fit into the above list.

The many-state nature of the variables of the model seems to preclude a numerical study by the transfer matrix method, because many-state models normally have a transfer matrix whose dimension grows as s^L, where s is the number of states taken on by the spin variables and L is the lattice length. However, it is simple to see that the dimension of the transfer matrix for RSOS models is not so large, due to the constraint of neighbouring sites differing by one. One can derive an upper bound by going to an arbitrary one-dimensional lattice configuration, and putting any odd number from 1 to $r-1$ on the leftmost site. Then, on each other site, put $+1$ if the site variable differs from its left neighbour by 1, or -1 if it differs by -1. Then an upper bound to the dimension of the transfer matrix is given by

$$B=\tfrac{1}{2}(r-1)2^{(L-1)}. \tag{8}$$

3. Numerical analysis of the III–IV transition

In this section, I estimate the conformal anomaly associated with the critical point of the RSOS models in the III–IV regime, for values of r up to 12. If the conjecture of Huse is correct, the values of c should fall into those given by equation (5), with $r=m-1$. For $r=4, 5$, the universality classes are those of the Ising and three-state Potts model; the validity of the Huse conjecture has been verified in these cases both numerically and analytically [6].

To compute the conformal anomaly, I use the technique of [6], where finite-size scaling of the free energy leads directly to c. At the critical point, for a system with periodic boundary conditions, the free energy should behave as

$$F/L=f_\infty-\pi c/6L^2+\mathrm{O}(1/L^4) \tag{9}$$

where f_∞ is the infinite-lattice result. To extract the approximation to c for a given lattice size, $c(L)$, we use a 'two-point fit':

$$c(L)\equiv\frac{6L(L-2)(F(L)-F(L-2))}{\pi[L^2-(L-2)^2]}. \tag{10}$$

A more detailed description of the numerical work is contained in the appendix.

In table 1, the extrapolated [7] values for c are given. These extrapolated values are in excellent agreement with the conjecture of Huse that this transition represents

Table 1. Measured and exact values for c.

r	c_{extr}	c_{exact}
4	0.500 08 ± 0.0001	0.5
5	0.700 ± 0.001	0.7
6	0.800 ± 0.001	0.8
7	0.857 ± 0.001	0.857
8	0.892 ± 0.001	0.893
9	0.917 ± 0.002	0.917
10	0.932 ± 0.002	0.9333
11	0.944 ± 0.003	0.945
12	0.954 ± 0.003	0.955

an example of all of the multicritical points in the FQS classification. It should be noted that the fact that the models are unitary, with c less than 1, together with the fact that c is determined accurately enough to identify it uniquely in the FQS series, provide a numerical proof of the Huse conjecture for r between 4 and 12.

Before concluding this section, I will look at one more question related to the III–IV transition—the universality of amplitudes in finite-size scaling as the parameter v, which measures the anisotropy of the couplings, is varied. Confidence in this universality will be useful in the next section, where the (anisotropic) antiferromagnetic transition is studied, and where formula (9) is not applicable because conformal invariance is no longer present. In [8] the scale dimensions of the operators of the quantum field theory are related to finite-size scaling amplitudes at a conformally invariant critical point

$$1/\xi = 2\pi x/L + \text{corrections}. \tag{11}$$

Here ξ is the correlation length associated with the operator, x is the scale dimension of the operator, and the falloff of the corrections is governed by the other operators in the theory. To verify equation (11) for the RSOS models, I give, in table 2, the (one-point) approximations to the lowest scale dimensions of the RSOS models for $n = 0$, obtained from the first gap in the spectrum of the transfer matrix

$$x(L) = L/2\pi\xi. \tag{12}$$

Table 2. Measured and exact values for x.

r	x_{extr}	x_{exact}
4	0.999 95 ± 0.0001	1.0
5	0.074 5 ± 0.02	0.075
6	0.133 8 ± 0.001	0.133 3
7	0.036 0 ± 0.001	0.035 7
8	0.075 ± 0.005	0.071
9	0.019 ± 0.003	0.020 8
10	0.045 ± 0.002	0.044
11	0.014 6 ± 0.001	0.013 7
12	0.031 ± 0.001	0.030 3

While the convergence for x is not as good as for c, the numbers are still in good agreement.

Neither this amplitude, nor the amplitude defined in the finite-size scaling of the free energy, remain unchanged as the parameter v is changed. It is expected, however, that the ratio of these amplitudes remains unchanged for large lattices. It is simple to see why this is the case by considering the transfer matrix formulation of the model. Anisotropy means that the correlation length in the 'time' direction differs from that in the 'space' direction. But isotropy can be restored by changing the scale of time and this scale can be changed by taking powers of the transfer matrix. For example, to rescale the 'time' by a factor of two, we replace T by $T^{1/2}$. Transforming to a diagonal basis for T, we see that anisotropy can be combined with conformal invariance by multiplying all energies (and hence all amplitudes) by some unknown constant. Thus ratios of amplitudes should be universal in the usual sense. This has been noticed previously for Hamiltonian formulations [9, 10]. In figure 2, I plot, for the Ising model in the RSOS formulation, values of the ratio $c(L)/x(L)$ for several values of v, in the III–IV regime, for lattice sizes up to $L = 12$.

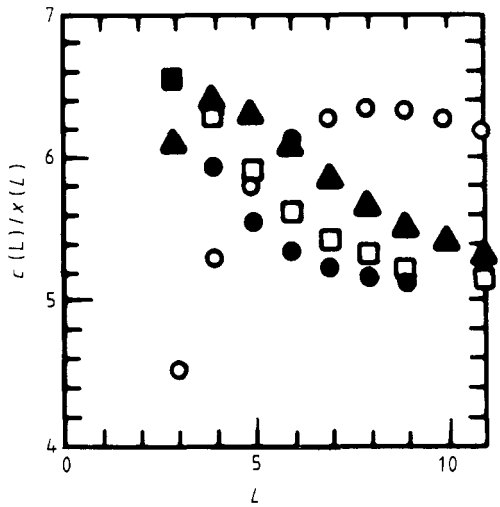

Figure 2. Convergence of $c(L)/x(L)$ for $r = 4$ for various values of $v = 0$ (●), 0.1 (□), 0.19 (▲), 0.3 (○).

It can be seen from these results that convergence suffers as a result of adding anisotropy although the estimates are consistent with convergence to the exact result. This lack of convergence is due to the marginal nature of the anisotropy operator [11]. A marginal operator in general provides logarithmic corrections to scaling which are the subject of a recent paper by Cardy [5] where the scaling field for the marginal operator is assumed to be small. Combining my comments concerning rescaling of the transfer matrix with the work of [5], I predict that

$$\frac{1}{\xi} = \gamma L^{-1}\Phi\frac{g_0}{4\pi b g_0 \ln L} + O(g^3) \tag{13a}$$

$$c(L) = \gamma(c + 4\pi^3 b g(L)^3) + O(g^4) \tag{13b}$$

where

$$g(L) = \frac{g_0}{1 + \pi b g_0 \ln L} \tag{14}$$

with $b, \Phi(x)$ universal and g_0 some unknown constant.

I test this prediction in the $r = 4$ model with the following algorithm.

(1) Use (13*b*) with g_0 large (so that g_0 dependence is removed) to predict b.

(2) Use (13*a*) on the large lattices to determine γ.

(3) Determine $g(L)$ from (13*b*).

(4) For each value of v, plot $x(g(L))$. All of these plots should fall on the same curve.

The plot obtained in the final step in this algorithm is shown in figure 3. Consistency with the above prediction is obtained when the anisotropy is not too large, where one can expect significant corrections from $O(g^3)$. Also, it should be noted that the lattice sizes for $r = 4$ have been divided in half because in this model the even sites are frozen at 2 and do not affect the critical properties.

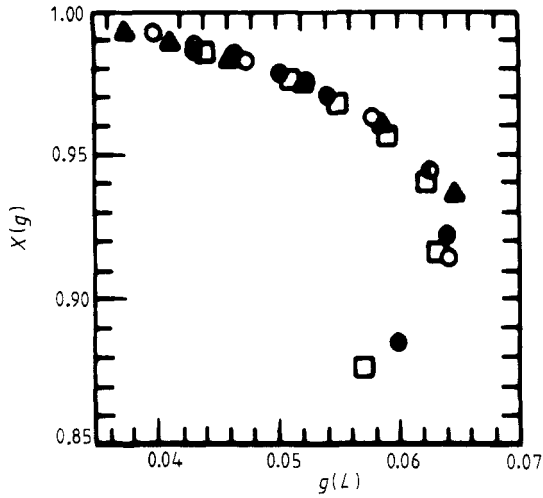

Figure 3. Universality of corrections to scaling for the $r = 4$ model. Lattice sizes range from 2 to 11, with larger lattices in the upper left-hand corner. $v = 0.1$ (▲), 0.15 (○), 0.19 (●), 0.2 (□).

4. Numerical analysis of the (antiferromagnetic) I–II transition

The numerical study of the antiferromagnetic transition is much more difficult than that of the III–IV transition for two reasons.

(i) The ground state of the model has a large period [3] (figure 4). This means that, for periodic boundary conditions, one only expects amplitude estimates to converge when using lattice sizes which are multiples of the period of the ground state. In table 3, I give an example of the finite-size amplitudes for $r = 5$. A similar phenomenon occurs for higher r but with larger period. Clearly models with $r \geq 5$ cannot be studied using this method.

3 4 5 6 5 4 3 2 [1 2 3 4 5 6 5 4 3 2] 1 2 3 4 5 6 5 4 3 2 1 2
4 5 6 5 4 3 2 1 2 3 4 5 6 5 4 3 2 1 2 3 4 5 6 5 4 3 2 1 2 3
5 6 5 4 3 2 1 2 3 4 5 6 5 4 3 2 1 2 3 4 5 6 5 4 3 2 1 2 3 4
6 5 4 3 2 1 2 3 4 5 6 5 4 3 2 1 2 3 4 5 6 5 4 3 2 1 2 3 4 5
5 4 3 2 1 2 3 4 5 6 5 4 3 2 1 2 3 4 5 6 5 4 3 2 1 2 3 4 5 6
4 3 2 1 2 3 4 5 6 5 4 3 2 1 2 3 4 5 6 5 4 3 2 1 2 3 4 5 6 5

Figure 4. Ground state for the $r = 7$ model, where one period is given in the box. Note that the NE-SW diagonal has a uniform ground state.

Table 3. Illustration of finite-lattice approximations to c, comparing the row-to-row and diagonal transfer matrix for $r = 5$ in the antiferromagnetic phase. Similar things happen for $r = 4$ and higher r.

L	$c(L)_{\text{row-row}}$	$c(L)_{\text{diagonal}}$
6-12	0.684 87	1.4610
12-18	0.699 9	1.5308
4-8	−0.718 418	1.2948
8-10	−0.677 734	1.5066
10-14	−0.698 931	1.5407
6-8	−2.455 522	1.4199
8-12	−1.793 976	1.5184

(ii) The model is anisotropic. This means that, judging from the results of the previous section, we can expect problems with convergence due to the presence of a marginal operator.

These problems can, however, be solved. To circumvent the first problem, I use a *diagonal-to-diagonal* transfer matrix [12] which is explained in figure 5. The basic idea is that the 'time' direction of the transfer matrix is rotated by 45°. As shown in figure 5, this ensures that the ground state fits on the 'space' direction of the transfer

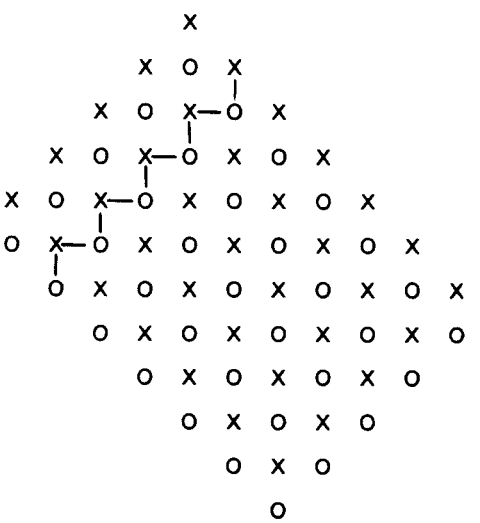

Figure 5. Diagonal-to-diagonal transfer matrix for the $r = 4$ model. × and ○ represent up and down spins, respectively. The lines join spins in one spatial row.

matrix for any r. Using this trick, the problem of the ground state disappears, as shown by the estimates for $r = 5$ in table 4.

As for the second problem, the anisotropy of the model slows drastically the convergence of the lattice estimates; also, because the size of lattice I can use is not particularly large for values of r greater than 5, I cannot fit logarithmic corrections as in § 4. However, I *can* vary the parameter v to minimise the corrections due to anisotropy. In this application there is a conjecture for the central charge and so one can simply vary v until the variable γ in equation (13*b*) is one, which indicates that the associated conformal field theory is isotropic. For general applications, one would vary the amount of isotropy until the best convergence was found. In figure 6, I give a comparison of convergence for two different values of γ for $r = 5$. As can be seen, the convergence does improve greatly as γ approaches 1.

With these comments in mind, I present table 4, which gives estimates of the central charge of the RSOS model for r between 4 and 9. These numbers were obtained by using the diagonal-to-diagonal transfer matrix and searching the space of v until isotropy prevailed. Some of the estimates of c for r greater than 6 have no error ranges

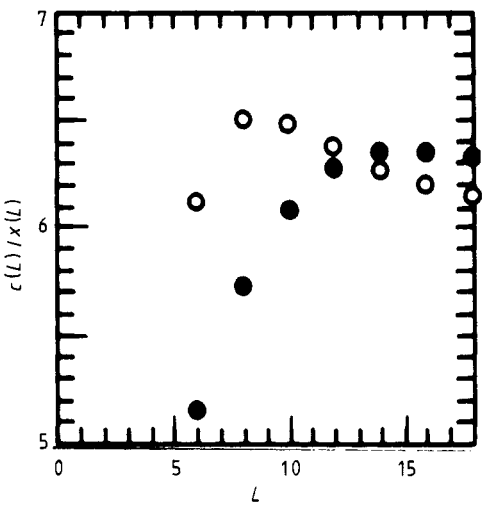

Figure 6. Comparison of convergence of finite-lattice estimates of c/L for two different magnitudes of the isotropy γ. ●, $v = 0.624$, $\gamma = 2.0$. ○, $v = 0.84$, $\gamma = 1.0$. A γ of one represents an isotropic model.

Table 4. Estimates for the ratio of central charge to scale dimension of the antiferromagnetic transition for $r = 4$–9. Estimates without error ranges are simply the values found on the largest lattices studied.

r	$(c/x)_{est}$	$(c/x)_{conj}$
4	$0.499\,95 \pm 0.0001$	0.5
5	6.04 ± 0.1	6
6	5.95 ± 0.2	6
7	$\simeq 10.7$	10
8	7.6 ± 0.6	7.5
9	$\simeq 14.7$	14

and are meant only as indications of the asymptotic values. The reason for this is that, for r greater than 6, even for the 'best' value of v, convergence of the lattice estimates for c was very poor. The numbers without error ranges quoted in table 4 are simply the values taken on the largest lattices sampled. For $r = 4, 5, 6, 8$ the data are consistent with the conjecture that these models are in the universality class of the Z_n transition with $r = n + 2$.

5. Conclusions

In this paper I have studied various aspects of conformal invariance in the RSOS models of Andrews *et al* [2]. For the ferromagnetic transition, for $r < 13$, a numerical proof has been given of the conjecture of Huse [3] that these models give examples of the generic multicritical points of Freidan *et al* [1].

I have also given evidence that, for $r = 4, 5, 6, 8$, the antiferromagnetic transition falls into the universality class of Z_n, with $n = r - 2$, as is expected from our partial knowledge of the scale dimensions. For the other values of r studied, I was unable to obtain sufficient convergence on the lattice studied to determine the universality class, but the estimates of the central charge of the Virasoro algebra obtained from the largest lattices falls reasonably close to the value expected from the conjecture that these models are also in the Z_n universality class. My data indicate that the models are conformally invariant and have $c > 1$.

I have also shown that finite-size scaling can be used to study anisotropic systems if logarithmic corrections due to the marginality of the anisotropy field are taken into account. This idea is summarised in figure 3.

Finally, it would be interesting to formulate a Hamiltonian limit of the model for at least two reasons. First, one could study the effect of anisotropy on the convergence of lattice estimates in this limit. Second, larger lattices are possible in the Hamiltonian version and better results might be obtained for the antiferromagnetic transition.

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Appendix. Some numerical details

In this appendix I will briefly explain how the data presented in the text were obtained. The first choice to be made was of the method used to diagonalise the transfer matrix. Two programs were written. The first simply stored the transfer matrix in memory and used a conventional algorithm for diagonalisation, taking into account the fact that diagonalisation in a translationally invariant subspace is sufficient. The second used a Lanczos method [13] which required only a small number of vectors, of length equal to the dimension of the transfer matrix, to be stored. Here one applies the

transfer matrix many times to an initial vector, generating a new matrix which has very low dimension (about twenty) and has identical low-lying eigenvalues to the original matrix. The vector obtained at each step was kept in two lists which were kept sorted so that they could be searched in a binary fashion. Also, the transfer matrix was factorised into $L/2$ sparse matrices, where L was the spatial length of the lattice. For large-dimension transfer matrices, the first method should go like d^2 and the second like $d \ln d$ where d is the dimension of the transfer matrix and L the lattice length, making the second method faster than the first (as well as more compact). Unfortunately, the transfer matrix used here has a rather complicated form, with next-nearest-neighbour interactions, which gave a significant advantage to the first naive method. The asymptotic computational effort mentioned above never set in on the lattices studied, which included transfer matrices of dimension up to 10^4. Thus all of the data given in the text were generated using the naive method. Much of it was checked using the independent Lanczos program. In table 5, I show some of the timings and storage requirements for sample values of r on large lattices.

Table 5. Timings and storage requirements for values of r studied using the naive method. CPU time is given for a Ridge-32 minicomputer. Storage requirements are determined by the dimension of the transfer matrix, dim T, in the translationally invariant subspace.

r	L_{max}	dim T	CPU time (min)
4	12	434	550
5	18	546	600
6	16	424	641
7	14	551	300
8	14	398	402
9	14	1042	1380
10	14	644	1020
11	12	530	230
12	14	890	1900

Next, I discuss the error analysis for the data presented in the text. For the extrapolated values for c and x presented in tables 1 and 2, the data were sufficiently convergent to apply the method of VBS sequence transformations, as described in [14], for both the extrapolated measurements and the error ranges. For the data shown in figures 2, 3 and 6, the only source of error was computer roundoff error. Double precision was used in all computations, and thus the error bars in these figures would be much too small to plot on the scales shown. Note that in figure 3, a source of error would be introduced in the determination of γ. This was removed by shifting each curve up or down to get the maximum amount of superposition.

In table 4, the extrapolations and error ranges for $r = 4$ and $r = 5$ were generated using the VBS method. For $r = 7$ and $r = 9$, no error estimates were made, because the data did not converge and are included only to show that the lattice estimates were in the right ballpark. For $r = 6$ and $r = 8$, the estimates seemed, at large lattice sizes, to be converging to the numbers given in table 4, but not enough numbers in the asymptotic region were obtained to form reliable VBS approximants. The lattice estimates here were extrapolated by eye, and the error ranges are a subjective estimate of the error involved in this extrapolation.

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